

## EXPERIMENT 5

**A) EXPERIMENT NAME:** Three Point & Four Point Bend Test (Flexure Test)

**B) THE AIM OF THE EXPERIMENT:**

The bend (flexure) test method measures behavior of materials subjected to simple beam loading. It is also called a transverse beam test with some materials. The test is used to measure the Young's modulus of a material.

**C) EXPERIMENTAL SETUP AND APPARATUS**

The testing equipment is Shimadzu Autograph AG-IS 100 computerized servo hydraulic universal testing machine (UTM) (Figure 1). The tensile tests are performed according to ASTM D7264 / D7264M - 15 "Standard Test Method for Flexural Properties of Polymer Matrix Composite Materials". This standard specifies the method for determining the ability of polymer matrix composite materials to undergo plastic deformation in bending. Applies to the bend test of test pieces taken from composite materials as specified in the relevant standard.

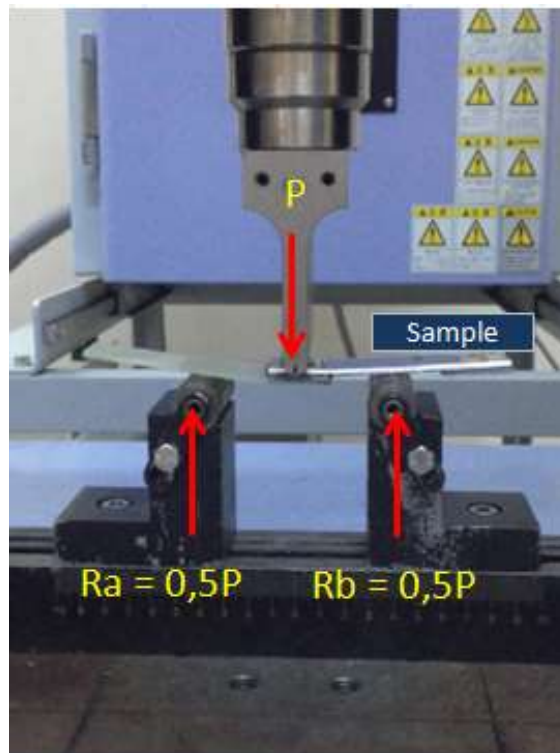


Figure 2. A photograph of a tensile machine with bending apparatus (Shimadzu Autograph)

## D) THEORY

If a beam is simply supported at the ends and carries a concentrated load at the center, the beam bends concave downwards. The distance between the original position of the beam and its position after bending is different. This difference is called ‘deflection’. In the case which occurring bending, takes places maximum deflection at the center along the length.

In the flexure (bend) test, maximum stress and maximum strain are calculated for increments of load. Results are plotted in a stress-strain diagram. Flexural strength is defined as the maximum stress in the outermost fiber. This is calculated at the surface of the specimen on the convex or tension side. Flexural modulus is calculated from the slope of the stress vs. deflection curve. If the curve has no linear region, a secant line is fitted to the curve to determine slope.

There are two test types; three point bent test four point bend test. In a three point test the area of uniform stress is quite small and concentrated under the center loading point. In a four point test, the area of uniform stress exists between the inner span loading points (typically half the outer span length).

The three point bend test (Figure 2) is a classical experiment in mechanics, used to measure the Young’s modulus of a material in the shape of a beam. The beam, of length  $L$ , rests on two roller supports and is subject to a concentrated load  $P$  at its centre (Figure 2).

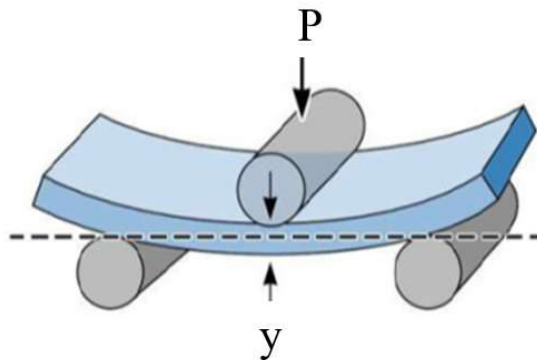


Figure 2. The three point bend test

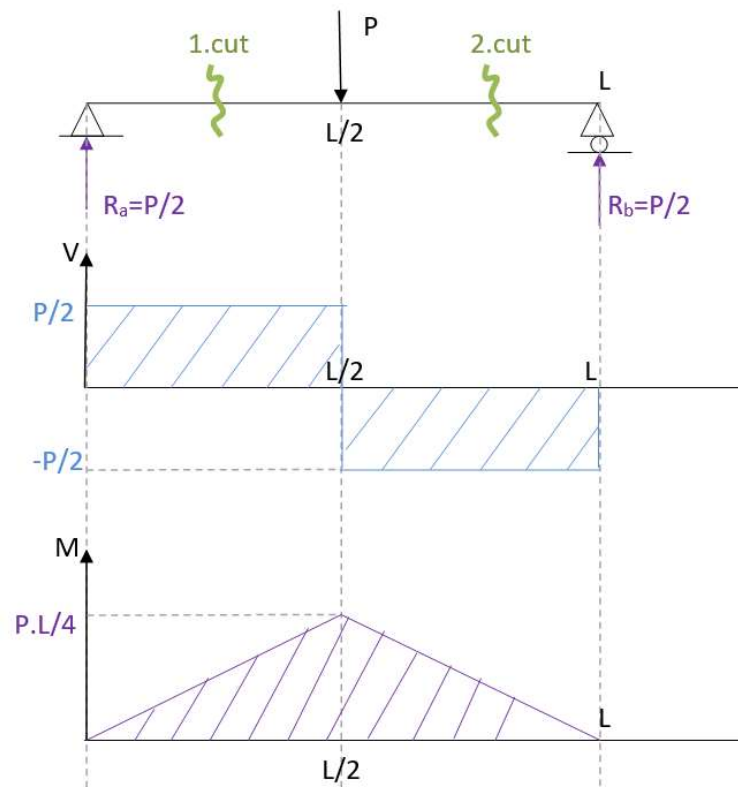
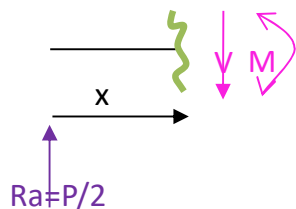


Figure 3. Schematic of the three point bend test (top), with graphs of bending moment  $M$ , shear force  $V$

The maximum deflection  $y_{\max}$  at the centre of the beam can be shown by the following deflection analysis (Figure 3):

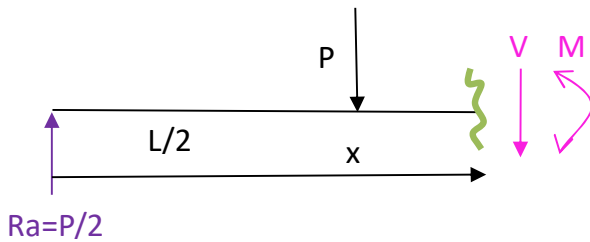
**1. cut:**



$$\begin{aligned} \sum F = 0 & \longrightarrow \frac{P}{2} - V = 0 \\ & V = \frac{P}{2} \\ \sum M = 0 & \longrightarrow \frac{P}{2}x - M = 0 \end{aligned}$$

$$M = \frac{P}{2}x \xrightarrow{x=L/2} M = \frac{PL}{4}$$

**2. cut:**



$$\sum F = 0 \longrightarrow \frac{P}{2} - P - V = 0$$

$$V = \frac{-P}{2}$$

$$\sum M = 0 \longrightarrow \frac{P}{2}x - P\left(x - \frac{L}{2}\right) - M = 0$$

$$M = \frac{PL}{2} - \frac{Px}{2} \xrightarrow{x=L/2} M = \frac{PL}{4}$$

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

$$EI \frac{d^2y}{dx^2} = M$$

**1. cut:**

$$EI \frac{d^2y}{dx^2} = \frac{Px}{2}$$

$$EI \frac{dy}{dx} = \frac{Px^2}{4} + c_1$$

$$(x = \frac{L}{2}, \frac{dy}{dx} = 0; c_1 = -\frac{PL^2}{16})$$

$$EI y = \frac{Px^3}{12} + c_1 x + c_2$$

$$(x=0, y=0; c_2 = 0)$$

$$y = \frac{Px^3}{12EI} - \frac{PL^2}{16EI} x$$

$$\text{For } x = \frac{L}{2},$$

$$y_{max} = \frac{PL^3}{96EI} - \frac{PL^3}{32EI} = -\frac{PL^3}{48EI}$$

**2.cut:**

$$EI \frac{d^2y}{dx^2} = \frac{PL}{2} - \frac{Px}{2}$$

$$EI \frac{dy}{dx} = \frac{PLx}{2} - \frac{Px^2}{4} + c_1$$

$$(x = \frac{L}{2}, \frac{dy}{dx} = 0; c_1 = -\frac{3PL^2}{16})$$

$$EI y = \frac{PLx^2}{4} - \frac{Px^3}{12} + c_1 x + c_2$$

$$(x=L, y=0; c_2 = \frac{PL^3}{48})$$

$$y = \frac{PLx^2}{4EI} - \frac{Px^3}{12EI} - \frac{3PL^2x}{16EI} + \frac{PL^3}{48EI}$$

$$\text{For } x = \frac{L}{2},$$

$$y_{max} = \frac{PL^3}{16EI} - \frac{PL^3}{96EI} - \frac{3PL^3}{32EI} + \frac{PL^3}{48EI} = - \frac{PL^3}{48EI} \quad (1)$$

where

P = Load acting at the center (N)  
L = Length of the beam between the supports (mm)  
E = Young's modulus of material of the beam (MPa)  
M = Bending moment (N.mm)

and

*I* is the second moment of area for given crosssection (Figure 4) defined by

$$I = \frac{bh^3}{12} \quad (2)$$

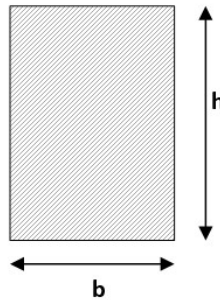


Figure 4. The beam cross section

Where ***h*** is the beam's height and ***b*** is the beam's width. By measuring the central deflection  $y_{max}$  and the applied force P, and knowing the geometry of the beam and the experimental apparatus, it is possible to calculate the Young's modulus of the material.

If the applied force P is plotted against central displacement y, a straight line is obtained. The gradient of this line is

$$\frac{dP}{dy} = \frac{48EI}{L^3} \quad (3)$$

### E) EXPERIMENTAL PROCEDURE:

1. Measure the width and thickness of the specimen.
2. Mark on the locations where the load will be applied under three-point bending. Note the length of between support.
3. Place the sample carefully on to the stage of 3-point bending fixture of a universal testing machine
4. Make sure that the loading point is placed on to the marked location.
5. Carry out the bend test

### F) ASSIGNMENTS

1. Calculate moment of inertia of bend sample section.
2. Calculate maximum deflection, moment and stress from the given formulas.
3. Draw the load-deflection graph to calculate the flexural bend strength and elastic modulus of the specimen.
4. On the graph choose any two convenient points in elastic area and between these points find the corresponding values of P and y.
5. Calculate Young's modulus by substituting P and y values in the equation 4.

$$E = \frac{PL^3}{48 yI} \quad (4)$$

6. Draw force and moment diagram then drive the deflection equations for 4 point bending test specimen (Figure 5 )

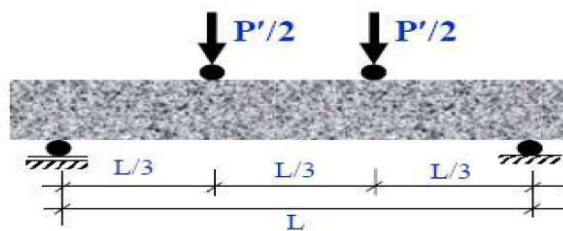


Figure 5. 4 point bending test